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ON PLANE NUCLEAR VORTEX

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We analyze the pure vortical motion in nuclear systems excluding the usual approximation on smallness of the excitation amplitude and the additional assumptions on the shape of a nuclear system. The equations to describe a nonlinear plane nuclear vortex are presented in the frame of nuclear hydrodynamics. The evolution of the shape of a vortex is analogous to a propagation of a nonlinear dispersion wave in a plane. These states can be considered as a generalization of elliptic Kirchhoff vortex. We have proved that the solutions having the symmetry relative to the turn by an angle $2\pi/l$ with integer parameter $l = 2, 3, 4\ldots$ can exist. These vortexes seem to be two-dimensional analogues to the rotating nuclear systems having stable quadrupole, octupole, hexadecupole deformations accordingly.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

О плоском ядерном вихре

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Чисто вихревое движение в ядерных системах анализируется без использования обычных приближений о малости амплитуды возбуждения и добавочных предположений о форме ядерной системы. В рамках ядерной гидродинамики представлены основные уравнения для описания плоских нелинейных ядерных вихрей. Эволюция формы вихря аналогична распространению в плоскости нелинейной дисперсной волны. Эти состояния могут быть рассмотрены как обобщение эллиптических вихрей Кирхгофа. Показано, что могут существовать решения, имеющие симметрию относительно поворота на угол $2\pi/l$ с целым параметром $l = 2, 3, 4\ldots.$ Эти вихри могут быть двумерными аналогами вращающихся ядерных систем, имеющих соответственно стабильную квадрупольную, октупольную, гексадекапольную... деформации.

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1. Introduction

Rotating states were always in the focus of attention of theoretical and experimental physics [1]. High spin states (e.g., see [2] and reference therein) is the most popular type of the vortical motion in nuclear systems, but

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far from being the only possible one. Many attempts were undertaken to find topological nontirival vortical states. Vortical isomer nuclei were supposed to exist [3] (the superconduction component of the nuclear fluid with the quantum curl was oriented along the symmetry axis of a rotating drop). Not long ago a supposition close in the physical sense to the above was mentioned again [4]. It was shown that superfluidity caused by triplet Cooper pairing makes the nuclear liquid anisotropic and also leads to discrete rotational spectra at high excitation energies [5]. The approximate solution to describe stable vortexes was obtained in the framework of the liquid drop model. In the first case, the solutions corresponded to the «hot spot», emerged in peripheral collisions [6]. In the second case, the soliton type solutions on a nuclear surface were associated with a cluster type configuration [7], [8]. The analogy with the admixture electrons and positrons in dense gases [9], [10] allows us to suppose the existence of the vortical «rings» on admixture hadrons in nuclear systems.

Recently, nuclear theory has predicted the formation of the new exotic vortical objects such as «disks» [11], [12], unstable hollow «bubbles» and «rings» [13] which decay by intermediate mass emission.

In this report we analyse the pure vortical motion excluding the usual approximation on smallness of the excitation amplitude and the additional assumptions on the shape of the nuclear system.

In Sec.2 the basic equations to describe plain nuclear vortex are presented within the nuclear hydrodynamics. Qualitative analysis of the main features of a vortes is done in Sec.3. Symmetry of the solutions is considered in Sec.4. It is shown that there probably exists the possibility to derive the stable solutions having the symmetry relative to the turn by an angle $2\pi/l$ with the integer parameter $l = 2, 3, 4\ldots$ Last Section contains a short symmary.

2. Basic Equations

We use the semiclassical nuclear hydrodynamics based on the current $\hat{j}$ and density $\hat{\rho}$ algebra and hydrodynamic representation for the nuclear hamiltonian, which is equivalent in view of the equations of motion for $\hat{j}$ and $\hat{\rho}$ to the initial hamiltonian. Gradient terms of the «pressure» drop out from the equations of motion on separating the curl component of the velocity field and the equations of motion for rot v are formally reduced to the pure kinematic form, at least for the Skyrme type forces.

For an incompressible ($\rho \equiv \rho_0$) nuclear vortical flow it is convenient to turn to the vorticity ζ and the vector potential A

$$\mathbf{v} = \mathrm{rot}\,\mathbf{A}, \qquad \mathrm{div}\,\mathbf{A} = 0, \qquad \frac{\partial}{\partial t}\zeta + v_r\frac{\partial}{\partial r}\zeta + v_\phi\frac{1}{r}\frac{\partial}{\partial \phi}\zeta = 0,$$

$$\zeta = \text{rot } \mathbf{v} = \text{rot rot } \mathbf{A} = \text{grad div } \mathbf{A} - \Delta\mathbf{A} = -\Delta\mathbf{A}.$$

We restict ourselves to the simplest rotational flow, two-dimensional motion $v(r, \phi) = v_r \mathbf{e}_r + v_\phi \mathbf{e}_\phi$, $\mathbf{A} = A\mathbf{e}_z$, $\zeta = \zeta\mathbf{e}_z$, where (r, ϕ) are polar coordinates of a point. The velocity projections v_r, v_ϕ can be determined by differentiating $A(r, \phi)$ with respect to r and ϕ: $v_r(r, \phi) = r^{-1}\partial A/\partial\phi$, $v_\phi(r, \phi) = -\partial A/\partial r$. The current function $A(r, \phi)$ can be derived from the Poisson equation by the two-dimensional Green function for the Laplace operator $A(r, \phi) = (2\pi)^{-1}\int d\phi' dr' r' \ln(|\mathbf{r} - \mathbf{r}'|)\,\zeta(r', \phi')$.

In this report we consider two-dimensional analogues of the nuclear «disks» — plane nuclear vortexes — a new type of a pure vortical state of incompressible nuclear matter. They are the finite areas of the constant vorticity on a plane $\zeta(r', \phi', t) \equiv \zeta_0$ within the uniform-rotating contour $\Gamma(r, \phi) \equiv r - R(\phi) = 0$. In essence, these states can be considered as the generalization of the elliptic Kirchhoff vortex [14]. The dynamical condition on the contour $(\mathbf{n}\cdot\mathbf{v}) = (\mathbf{n}\cdot\mathbf{v}_{contour})$ and the normal vector n are given by

$$\mathbf{n} = \frac{\sigma\nabla\Gamma}{|\nabla\Gamma|} = \sigma\,(\mathbf{e}_r - S(\phi)\,\mathbf{e}_\phi)\,(1 + S(\phi)^2)^{-1/2}, \qquad S(\phi) \equiv \frac{1}{R}\frac{dR}{d\phi},$$

$$\Omega\frac{dR}{d\phi} + v_r - v_\phi\left(\frac{1}{R}\frac{dR}{d\phi}\right) = 0,$$

where Ω is an angular velocity of the uniform-rotation of the contour and $\sigma = \pm 1$ defines the orientation of the contour.

Finally the equatin for the vortex boundary may be cast into the form of the following one-dimensional nonlinear integro-differential equation

$$\frac{2\pi\Omega}{\zeta_0}\frac{dR}{d\phi} = \int_0^{2\pi} d\phi' R((\phi')\ln(|\delta\mathbf{R}|)\,[(1 + S(\phi)\,S(\phi'))\sin(\phi' - \phi) +$$
$$+ (S(\phi) - S(\phi'))\cos(\phi' - \phi)],$$

where $|\delta\mathbf{R}| = (R(\phi)^2 + R(\phi')^2 - 2R(\phi)\,R(\phi')\cos(\phi - \phi'))^{1/2}$.

3. Qualitative Analysis

For a quantitative analysis of the Eq.(1) it is necessary to build its discrete analogue, that is, in progress. Here we shall present only the qualitative analysis which can be done by analogy with the well-known elliptic Kirchhoff vortex and the solution for small perturbations of the circle [14].

(i) Despite the internal part of the vortex is rotating with a constant angular velocity, this motion differs from the motion of a rigid body, as the contour is rotating with a different velocity, more slowly.

The small rotationless perturbation of a circle $\delta A(r, \phi) = \alpha (\zeta_0/2) R_0^2 (r/R_0)^l \cos(l\phi - \omega t)$, where l is an integer, gives us the following contour equation $R(\phi) = R_0(1 + \alpha \cos(l\phi - \omega t))$ (for $\alpha << R_0$). So the small perturbation given by trigonometrical functions, is a crimp moving along the circle vortex with the angular velocity $\Omega = \omega/l = (l - 1)\zeta_0/2l$. At $l = 2$ the perturbed shape is an ellipse, rotating about its center with the angular velocity $\zeta_0/4$, that is half of the velocity of the fluid into the contour. Perturbations of the higher symmetry $l \geq 3$ are rotating still slower.

(ii) The fixed ratio Ω/ζ_0 and the symmetry of states define completely the shape of the contour (for instance, for the elliptic vortex its eccentricity $\Omega/\zeta_0 = \varepsilon(1 + \varepsilon)^{-2}$.

Eq. (1) together with the definition of the velocity fields will describe the motion of the contour as the propagation of a nonlinear dispersion wave on a plane. At the beginning the moving contour will be inevitably distorted. However, if this state is stable, then the interference between the nonlinearity and the dispersion will lead to the return of the initial contour shape. If one could prove the existence of these states, then these vortexes will be an analogue of the solitons on a plane*.

(iii) The parameter Ω/ζ_0 will be the bifurcation parameter and will determine the vortex's stability. The integrals of motion are the square of the «disk» which is a two-dimensional analogue of the particle number and the circulation of the vortex defined by ζ_0. If the contour motion is unstable, one may expect the disintegration of the «disk» into the separate rotating vortexes and into the vortex filaments — two-dimension analogues of the rotating intermediate mass fragments.

In the next Sec. we will focus our attention on a symmetry of the solutions to Eq. (1).

4. Periodic Solutions

When $\alpha << R_0$, a contour velocity depends on the symmetry of a perturbation and the solutions may be classified by the parameter $l = 2, 3, 4...$, or by the symmetry relative to the turn by the angle $2\pi/l$. Here we shall show

*For a last decade history of a soliton concept in nonrelativistic nuclear physics see review [15].

that the solutions having a symmetry relative to the turn by the angle $2\pi/l$ can exist. We do not use any additional approximation on smallness of the excitation amplitude.

Let us prove that the periodic solutions of Eq. (1) can exist at the class of $C^1_{[0,2\pi]}$ functions.

Eq. (1) may be cast in the following form

$$\frac{\partial}{\partial\phi} R(\phi) = \int_0^{2\pi} d\phi' F(R(\phi), R(\phi'), \phi, \phi'). \tag{2}$$

The necessary condition for an existence of the periodic solutions

$$R(\phi + T) = R(\phi), \qquad T = 2\pi/l, \qquad l = 1, 2, 3\ldots \tag{3}$$

is the periodicity of the right-hand side of Eq. (2), i.e.

$$\int_0^{2\pi} d\phi' F(R(\phi + T), R(\phi'), \phi + T, \phi') = \\ = \int_0^{2\pi} d\phi' F(R(\phi), R(\phi'), \phi, \phi'), \tag{4}$$

because for the periodic function (3) the left-hand side of this equation is a periodic one

$$\frac{\partial}{\partial\phi} R(\phi + T) = \frac{\partial}{\partial\phi} R(\phi). \tag{5}$$

Let us prove the afore-said necessary condition. Let $R(\phi)$ be a solution of Eq. (2) with the property (3). Then from Eqs. (1), (3), (5) one has the equalities

$$S(\phi + T) = S(\phi), \quad |\delta\mathbf{R}|^2(\phi + T, \phi' + T) = |\delta\mathbf{R}|^2(\phi, \phi'). \tag{6}$$

Further the following obvious identities will be also needed

$$\sin(\phi + T + \phi') = \sin(\phi - (\phi' - T)), \\ \cos(\phi + T + \phi') = \cos(\phi - (\phi' - T)). \tag{7}$$

The left-hand side of Eq. (4) can be decomposed as the sum of the integrals

$$\int_0^{2\pi} \ldots = \int_0^{T} \ldots + \int_T^{2T} \ldots + \ldots + \int_{(2l-1)T}^{2\pi} \ldots \tag{8}$$

Let us consider the first one and reduce it to the other. Taking into account Eqs. (3), (5), (6) one has

$$\int_0^T d\phi' R(\phi')\,(1/2)\ln\left(R^2(\phi) + R^2(\phi') - 2R(\phi)\,R(\phi')\cos(\phi + T + \phi')\right)\times$$
$$\times\left(\sin(\phi + T + \phi')\,(1 + S(\phi)\,S(\phi')) + \right.$$
$$\left. + \cos(\phi + T - \phi')\,(S(\phi) - S(\phi'))\right). \qquad (9)$$

By replacement of the variables $t \equiv \phi' - T$, and $\phi = t + T$ integral (9) is written as

$$\int_{-T}^{0} dt(1/2)\ln\left(R^2(\phi) + R^2(t) - 2R(\phi)\,R(t)\cos(\phi - t)\right)\times$$
$$\times\left(\sin(\phi - t)\,(1 + S(\phi)\,S(t)) + \cos(\phi - t)\,(S(\phi) - S(t))\right). \qquad (10)$$

Analogously each integral $\int_{mT}^{(m+1)T} \ldots$ of the sum (8) is reduced to the following integral

$$\int_{(m-1)T}^{mT} d\phi' F(R(\phi), R(\phi'), \phi, \phi'), \qquad m = 1,\ldots, n-1. \qquad (11)$$

Thus the following equalities are proved for $m = 1,\ldots, n-1$

$$\int_{mT}^{(m+1)T} d\phi' F(R(\phi + T), R(\phi'), \phi + T, \phi') =$$
$$= \int_{(m-1)T}^{mT} d\phi' F(R(\phi), R(\phi'), \phi, \phi'). \qquad (12)$$

Consequently in order to prove the equality (4) one has to show that the following relation is fulfilled for the integral (10)

$$\int_{-T}^{0} dt\, R(t)\,(1/2)\ln\left((R^2(\phi) + R^2(t) - 2R(\phi)\,R(t)\cos(\phi - t)\right)\times$$
$$\times\left(\sin(\phi - t)\,(1 + S(\phi)\,S(t)) + \cos(\phi - t)\,(S(\phi) - S(t))\right) =$$
$$= \int_{(n-1)T}^{nT=2\pi} dt\, R(t)\,(1/2)\ln\left((R^2(\phi) + R^2(t) - 2R(\phi)\,R(t)\cos(\phi - t)\right)\times$$
$$\times\left(\sin(\phi - t)\,(1 + S(\phi)\,S(t)) + \cos(\phi - t)\,(S(\phi) - S(t))\right).$$

It is obvious because at a circle $R = R_0 =$ const an arc $-T \le \phi \le 0$ coincides with an arc $(n-1)\,T \le \phi \le 2\pi$.

So Eq. (4) ensues from Eq. (3).

Let us present the periodic solutions for the two utmost cases:

i) $n = 1$, then $R(0) = R(2\pi)$ and $R(\phi)$ is just a general solution.

ii) $n = \infty$, then $R(\phi) = R(\phi + \delta\phi)$ for any small $\delta\phi$, i.e. $R(\phi) = \text{const}$ is a circle.

Let us test that a circle $R = R_0$ is a solution of Eq.(1). One has $\frac{\partial}{\partial\phi} R = 0$, $S = 0$, and Eq.(1) turns a following equality which can be proved.

$$0 = \int_0^{2\pi} dt\, R_0(1/2)\ln(R_0^2 + R_0^2 - 2R_0^2\cos(\phi - t))\sin(\phi - t),$$

$$0 = \int_0^{2\pi} dt\ln(2R_0^2(1 - \cos(\phi - t))\sin(\phi - t),$$

$$0 = \int_0^{2\pi} dt\ln(2R_0^2)\sin(\phi - t) + \int_0^{2\pi} dt\ln(1 - \cos(\phi - t))\sin(\phi - t),$$

$$0 = \ln(2R_0^2)(\cos(\phi) - \cos(\phi - 2\pi)) + \int_0^{2\pi} dt\ln(1 - \cos(\phi - t))\sin(\phi - t).$$

Now it only remains to show that $\int_0^{2\pi} dt\ln(1 - \cos(\phi - t))\sin(\phi - t) = 0.$ Let us denote $u \equiv \cos(t - \phi)$, $v \equiv \ln(1 - u)$. Then one has

$$-\int_0^{2\pi} dtu'\ln(1 - u) = -u\ln(1 - u)\Big|_0^{2\pi} + \int_0^{2\pi} dt\, uu'(1 - u)^{-1} =$$

$$= -\int_0^{2\pi} dt\, uu'(1 - u)^{-1} = u\Big|_0^{2\pi} + \ln(1 - u)\Big|_0^{2\pi} = 0.$$

This is that we wanted to prove.

5. Summary

We analyze the pure vortical motion in nuclear systems excluding the usual approximation on smallness of the excitation amplitude and the additional assumptions on the shape of a nuclear system. The equations to describe a nonlinear plane nuclear vortex are presented in the frame of nuclear hydrodynamics. The evolution of the shape of a vortex is analogous to a propagation of a nonlinear dispersion wave in a plane. These states can be considered as a generalization of elliptic Kirchhoff vortex. A short qualitative analysis of the main features of the vortex is done.

We have shown that at the class of $C^1_{[0,2\pi]}$ functions the periodic solutions having the symmetry relative to the turn by an angle $2\pi/l$ with integer parameter $l = 2, 3, 4\ldots$ can exist and we have presented two such solutions. The expected vortexes are two-dimensional analogues to the rotating nuclear systems which have stable quadropole, octupole, hexadecupole deformations accordingly.

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